MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

ETN3096 – DIGITAL SIGNAL PROCESSING
(CE, EE, LE, OPE, TE)

5 MARCH 2020
2:30 PM – 4:30 PM
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 12 pages (including this cover page) with 4 Questions only and an appendix.
2. Attempt **ALL** questions. The distribution of the marks for each question is given.
3. Please print all your answers in the Answer Booklet provided.

Question 1

(a) What is aliasing? State the condition to ensure that a DSP system will not suffer from aliasing. [3 marks]

(b) Consider a discrete-time system $y[n] = ax^2[n+1]$. Determine, with justification, if the system is
(i) Linear [4 marks]
(ii) Time-invariant [4 marks]
(iii) Causal [2 marks]

(c) In the system shown in Figure Q1, $h_1[n] = [\ldots\ 0\ \ 5\ \ \underline{2}\ \ -1\ \ 1\ \ 0\ \ldots]$ and $h_2[n] = 2\delta[n] - \delta[n-2]$, where the underlined value corresponds to $n = 0$.

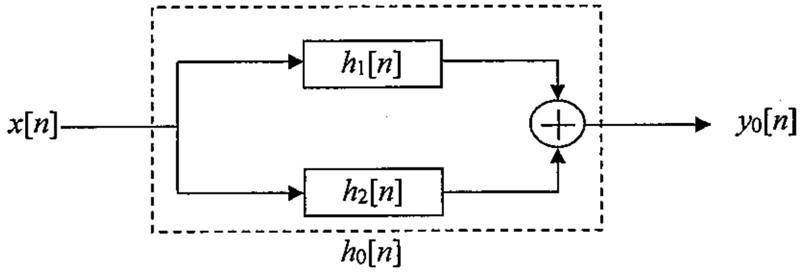


Figure Q1

(i) Determine the impulse response of the overall system $h_0[n]$. [3 marks]
(ii) For input $x[n] = 2n(u[n+1] - u[n-2])$, determine $y_0[n]$ for $n = -2$ to 3. [9 marks]

Continued ...

Question 2

(a) A causal linear time-invariant (LTI) system is described by a difference equation given by $y[n] = 0.7y[n-1] - 0.12y[n-2] + x[n] + x[n-1]$, where $x[n]$ is the system input and $y[n]$ is the output. All initial conditions are zero.

(i) Determine the system function $H(z)$. [2 marks]

(ii) Comment on the system stability. [3 marks]

(iii) Determine the system impulse response $h[n]$. [5 marks]

(b) (i) Compute the 4-point discrete Fourier transform (DFT) of $x[n] = [\underline{4} \ \ 2 \ \ 1 \ \ 0]$ using direct DFT. [9 marks]

(ii) Assume that a complex addition takes $1\mu s$ and a complex multiplication takes $2\mu s$. Compare the time required for the direct evaluation of a 256-point DFT and the time required for the evaluation of a 256-point DFT through fast Fourier transform (FFT). [6 marks]

Question 3

(a) Design a 5-tap finite impulse response lowpass filter with a cutoff frequency of 4000 Hz and a sampling rate of 10,000 Hz.

(i) Calculate the filter coefficients when a rectangular window function is used. [10 marks]

(ii) Determine the transfer function and difference equation of the filter. [6 marks]

(iii) Find the gain of the filter at dc (frequency = 0). [2 marks]

Continued ...

Question 3 (continued)

(b) Adaptive filters play an important role in modern digital signal processing applications.

(i) How does adaptive filter differ from conventional digital filter? [3 marks]

(ii) Give two advantages of adaptive filter over conventional digital filter. [4 marks]

(c) Consider the digital signal processing system for noise cancellation using an adaptive filter with two coefficients shown in Figure Q3. The weight update equation is given by

$$w_i(n) = w_i(n-1) + 2\mu e(n)x(n-i)\text{, for } i = 0, 1$$

Assume that $w_0(-1)=0$, $w_1(-1)=0$, $x(-1)=0$, and the convergence factor μ = 0.05. Apply adaptive filtering to obtain outputs $y(n)$ and $e(n)$ for n = 0, 1, 2, as well as filter weights $w_0(n)$ and $w_1(n)$ for n = 0, 1, given inputs $d(n)$ and $x(n)$ as shown in Table Q3. Copy the table in your answer sheet. Show your calculations. [8 marks]

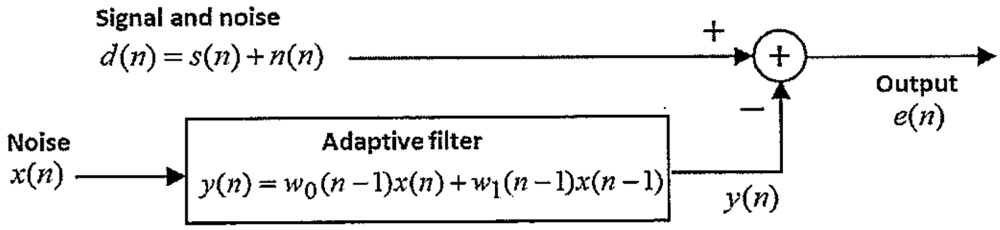


Figure Q3

Table Q3

Iteration n	Signal corrupted with noise $d(n)$	Noise signal $x(n)$	Filter output $y(n)$	Error signal $e(n)$	Filter weight $w_0(n)$	Filter weight $w_1(n)$
0	-1.5	-1				
1	-1	1				
2	1	-1				

Continued ...

Question 4

(a) The transfer function of an infinite impulse response (IIR) filter is given by

$$H(z) = \frac{(1-0.1z^{-1})(1-0.2z^{-1})(1-0.4z^{-1})}{(1-0.3z^{-1})(1-0.5z^{-1})(1-0.6z^{-1})}.$$

(i) Draw the Direct Form II realization of the filter. [8 marks]

(ii) For the realization in (i), evaluate the effects of quantizing the filter coefficients to 2 decimal places. (Note that no calculations are required here.) [4 marks]

(b) A finite impulse response (FIR) filter is described by its impulse response

$$h[n] = \frac{1}{n+1}(u[n]-u[n-3]).$$

Draw the Direct Form realization of the filter. [5 marks]

Continued ...

APPENDIX

Discrete-time Fourier transform

Properties

Property	$x[n], y[n]$	$X(e^{j\Omega}), Y(e^{j\Omega})$
Linearity	$ax[n] + by[n]$	$aX(e^{j\Omega}) + bY(e^{j\Omega})$
Time shifting	$x[n - n_0]$	$e^{-j\Omega n_0}X(e^{j\Omega})$
Frequency shifting	$e^{j\Omega_0 n}x[n]$	$X(e^{j(\Omega-\Omega_0)})$
Time reversal	$x[-n]$	$X(e^{-j\Omega})$
Differentiation	$n^k x[n]$	$(j)^k \frac{d^k}{d\Omega^k} X(e^{j\Omega})$
Convolution	$x[n] * y[n]$	$X(e^{j\Omega})Y(e^{j\Omega})$
Multiplication	$x[n]y[n]$	$\frac{1}{2\pi}(X(e^{j\theta}) * Y(e^{j\Omega}))$

Common pairs

$x[n]$	$X(e^{j\Omega})$
$\delta[n]$	1
$\delta[n - n_0]$	$e^{-j\Omega n_0}$
1	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\Omega + 2\pi k)$
$e^{j\Omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\Omega - \Omega_0 + 2\pi k)$
$u[n]$	$\frac{1}{1 - e^{-j\Omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\Omega + 2\pi k)$
$a^n u[n],\ \|a\| < 1$	$\frac{1}{1 - ae^{-j\Omega}}$
$(n+1)a^n u[n],\ \|a\| < 1$	$\frac{1}{(1 - ae^{-j\Omega})^2}$
$\frac{\sin\Omega_c n}{\pi n}$	$X(e^{j\Omega}) = \begin{cases} 1, & \|\Omega\| < \Omega_c \\ 0, & \Omega_c < \|\Omega\| \leq \pi \end{cases}$
$x[n] = \begin{cases} 1, & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$	$\frac{\sin\Omega\cdot(M+1)/2}{\sin\Omega/2} e^{-j\Omega M/2}$

z-transform

Properties

Properties	Sequence	z-transform	ROC
	$x[n]$ $x_1[n]$ $x_2[n]$	$X(z)$ $X_1(z)$ $X_2(z)$	R_x R_{x1} R_{x2}
Linearity	$ax_1[n]+bx_2[n]$	$aX_1(z)+bX_2(z)$	Contains $R_{x1} \cap R_{x2}$
Time shifting	$x[n-m]$	$z^{-m}X(z)$	R_x except for the possible addition or deletion of the origin or infinity.
Multiplication by an exponential sequence	$a^n x[n]$	$X(z/a)$	$\|a\|R_x$
Differentiation	$nx[n]$	$-z\dfrac{dX(z)}{dz}$	R_x except for the possible addition or deletion of the origin or infinity.
Conjugate	$x^*[n]$	$X^*(z^*)$	R_x
Time reversal	$x[-n]$	$X(z^{-1})$	$1/R_x$
Convolution	$x_1[n]*x_2[n]$	$X_1(z)X_2(z)$	Contains $R_{x1} \cap R_{x2}$

Common pairs

Sequence	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\dfrac{1}{1-z^{-1}}$	$\|z\| > 1$
$-u[-n-1]$	$\dfrac{1}{1-z^{-1}}$	$\|z\| < 1$
$\delta[n-m]$	z^{-m}	All z except 0 (if $m > 0$) or ∞ (if $m < 0$)
$a^n u[n]$	$\dfrac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
$-a^n u[-n-1]$	$\dfrac{1}{1-az^{-1}}$	$\|z\| < \|a\|$
$na^n u[n]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
$-na^n u[-n-1]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| < \|a\|$

Finite Impulse Response (FIR) Filters

Ideal impulse responses for standard FIR filters

Filter Type	Ideal Impulse Response $h(n)$
Lowpass:	$h(n)=\begin{cases}\dfrac{\Omega_c}{\pi}, & n=0\\ \dfrac{\sin(\Omega_c n)}{n\pi}, & n\neq 0\end{cases}$
Highpass:	$h(n)=\begin{cases}\dfrac{\pi-\Omega_c}{\pi}, & n=0\\ \dfrac{-\sin(\Omega_c n)}{n\pi}, & n\neq 0\end{cases}$
Bandpass:	$h(n)=\begin{cases}\dfrac{\Omega_H-\Omega_L}{\pi}, & n=0\\ \dfrac{\sin(\Omega_H n)-\sin(\Omega_L n)}{n\pi}, & n\neq 0\end{cases}$
Bandstop:	$h(n)=\begin{cases}\dfrac{\pi-(\Omega_H-\Omega_L)}{\pi}, & n=0\\ \dfrac{-\sin(\Omega_H n)+\sin(\Omega_L n)}{n\pi}, & n\neq 0\end{cases}$

FIR filter length estimation using window functions

Window Type	**Window Function** $w(n), -M\le n\le M$
Rectangular	1
Hanning	$0.5+0.5\cos\left(\dfrac{n\pi}{M}\right)$
Hamming	$0.54+0.46\cos\left(\dfrac{n\pi}{M}\right)$
Blackman	$0.42+0.5\cos\left(\dfrac{n\pi}{M}\right)+0.08\cos\left(\dfrac{2n\pi}{M}\right)$

Normalized Transition Width $\Delta f=\dfrac{\lvert f_{stop}-f_{pass}\rvert}{f_{sampling}}$

Type of Window	**Window Length N**	**Stopband Attenuation (dB)**	**Passband Ripple (dB)**
Rectangular	$N=0.9/\Delta f$	21	0.7416
Hanning	$N=3.1/\Delta f$	44	0.0546
Hamming	$N=3.3/\Delta f$	53	0.0194
Blackman	$N=5.5/\Delta f$	74	0.0017

Bilinear Transformation (BLT)

1. Frequency prewarping
Let ω_a denote the analog frequency marked on the $j\omega$ -axis on the s-plane, and ω_d denote the digital frequency marked on the unit circle in the z-plane.

For the lowpass filter and highpass filter:

$$\omega_a = \frac{2}{T}\tan\left(\frac{\omega_d T}{2}\right)$$

For the bandpass filter and bandstop filter:

$$\omega_{al} = \frac{2}{T}\tan\left(\frac{\omega_l T}{2}\right),\ \omega_{ah} = \frac{2}{T}\tan\left(\frac{\omega_h T}{2}\right)$$

and $\omega_0 = \sqrt{\omega_{al}\omega_{ah}}, W = \omega_{ah} - \omega_{al}$.

2. Prototype transformation using the lowpass prototype $H_P(s)$
From lowpass to lowpass: $H(s) = H_P(s)|_{s=\frac{s}{\omega_a}}$

From lowpass to highpass: $H(s) = H_P(s)|_{s=\frac{\omega_a}{s}}$

From lowpass to bandpass: $H(s) = H_P(s)|_{s=\frac{s^2+\omega_0^2}{sW}}$

From lowpass to bandstop: $H(s) = H_P(s)|_{s=\frac{sW}{s^2+\omega_0^2}}$

where ω_a denotes the analog frequency, $\omega_0 = \sqrt{\omega_{al}\omega_{ah}}, W = \omega_{ah} - \omega_{al}$.

3. Substitute the BLT to obtain the digital filter

$$H(z) = H(s)|_{s=\frac{2}{T}\frac{z-1}{z+1}}$$

Conversion from Analog Filter Specifications to Lowpass Prototype Specifications

Analog Filter Specifications	Lowpass Prototype Specifications
Lowpass: ω_{ap}, ω_{as}	$v_s = \dfrac{\omega_{as}}{\omega_{ap}}$
Highpass: ω_{ap}, ω_{as}	$v_s = \dfrac{\omega_{ap}}{\omega_{as}}$
Bandpass: $\omega_{apl}, \omega_{aph}, \omega_{asl}, \omega_{ash}$ $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}, \omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$v_s = \dfrac{\omega_{ash} - \omega_{asl}}{\omega_{aph} - \omega_{apl}}$
Bandstop: $\omega_{apl}, \omega_{aph}, \omega_{asl}, \omega_{ash}$ $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}, \omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$v_s = \dfrac{\omega_{aph} - \omega_{apl}}{\omega_{ash} - \omega_{asl}}$

ω_{ap}, passband frequency edge; ω_{as}, stopband frequency edge

ω_{apl}, lower cutoff frequency in passband; ω_{aph}, upper cutoff frequency in passband

ω_{asl}, lower cutoff frequency in stopband; ω_{ash}, upper cutoff frequency in stopband

ω_0, geometric center frequency

Closed-Form Expression for Some Useful Series

$\sum_{n=0}^{N-1} a^n = \dfrac{1-a^N}{1-a}$	$\sum_{n=0}^{\infty} na^n = \dfrac{a}{(1-a)^2}$
$\sum_{n=0}^{N-1} na^n = \dfrac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$	$\sum_{n=0}^{N-1} n^2 = \dfrac{1}{6}N(N-1)(2N-1)$
$\sum_{n=0}^{N-1} n = \dfrac{1}{2}N(N-1)$	$\sum_{n=N_1}^{N_2} a^n = \dfrac{a^{N_1-1} - a^{N_2}}{1-a}$
$\sum_{n=0}^{\infty} a^n = \dfrac{1}{1-a} \qquad \lvert a \rvert < 1$	

Digital Butterworth and Chebyshev Filter Designs

With the given passband ripple A_p dB at the normalized passband frequency edge $v_p = 1$, and the stopband attenuation A_s dB at the normalized stopband frequency edge v_s, ε is the absolute ripple specification

$$\varepsilon^2 = 10^{0.1A_p} - 1$$

Butterworth lowpass prototype order

$$n \geq \frac{\log_{10}\left(\dfrac{10^{0.1A_s} - 1}{\varepsilon^2}\right)}{2\log_{10}(v_s)}$$

Chebyshev lowpass prototype order

$$n \geq \frac{\cosh^{-1}\left(\sqrt{\dfrac{10^{0.1A_s} - 1}{\varepsilon^2}}\right)}{\cosh^{-1}(v_s)}$$

where $\cosh^{-1}(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$

3-dB Butterworth Lowpass Prototype Transfer Functions ($\varepsilon = 1$)

n	$H_p(s)$
1	$\dfrac{1}{s+1}$
2	$\dfrac{1}{s^2 + 1.4142s + 1}$
3	$\dfrac{1}{s^3 + 2s^2 + 2s + 1}$

Chebyshev Lowpass Prototype Transfer Functions with 1dB Ripple ($\varepsilon = 0.5088$)

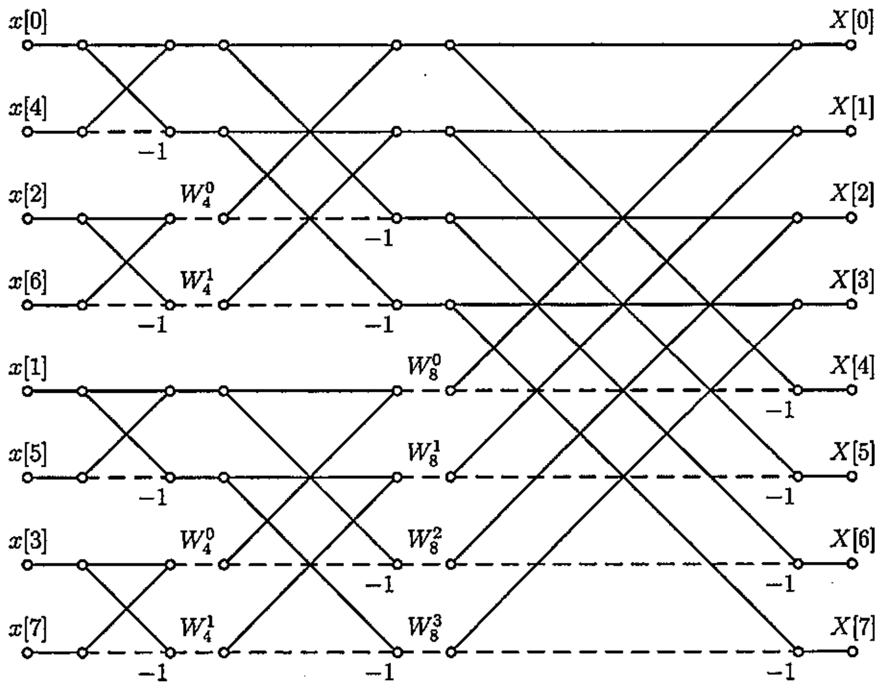
n	$H_p(s)$
1	$\dfrac{1.9652}{s + 1.9652}$
2	$\dfrac{0.9826}{s^2 + 1.0977s + 1.1025}$
3	$\dfrac{0.4913}{s^3 + 0.9883s^2 + 1.2384s + 0.4913}$

Discrete Fourier Transform

Properties

Property	$x[n]$	$X[k]$
Linearity	$A_1x_1[n] + A_2x_2[n]$	$A_1X_1[k] + A_2X_2[k]$
Time shifting	$x[\langle n - n_0\rangle_N]$	$X[k]W_N^{kn_0}$
Frequency shifting	$x[n]W_N^{-k_0n}$	$X[\langle k - k_0\rangle_N]$
Time reversal	$x[\langle -n\rangle_N]$	$X[\langle -k\rangle_N]$
Conjugation	$x^*[n]$	$X^*[\langle -k\rangle_N]$
Convolution	$x[n] \circledast y[n]$	$X[k]Y[k]$
Modulation	$Nx[n]y[n]$	$X[k] \circledast Y[k]$

The decimation-in-time fast Fourier transform



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